Karlsruhe Institute of Technology

Text Indexing

Lecture 12: Longest Common Extensions

Florian Kurpicz

Recap: Document Listing and Top-k Retrieval

Definition: Document Listing

Given a collection of D documents $\mathcal{D} = \{d_1, d_2, \ldots, d_D\}$ containing symbols from an alphabet $\Sigma = [1, \sigma]$ and a pattern $P \in \Sigma^*$, return all $j \in [1, D]$, such that d_j contains P.

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- $d_1 = \text{ATA}$
- $d_2 = \text{TAAA}$
- $d_3 = \text{TATA}$

And for queries:

- $P = \text{TA}$ is contained in d_1, d_2, and d_3
- $P = \text{ATA}$ is contained in d_1 and d_3

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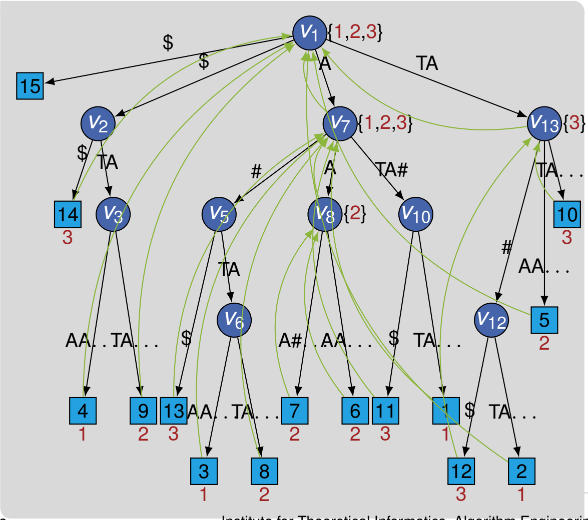

Recap: Inverted Index and List Encodings

Definition: Inverted Index

Given a set of documents and terms that are contained in the documents, an inverted index stores the terms and associated with each term t

- the number of documents f_t that contain t and
- an ordered list $L(t)$ of documents containing t

```
1 The old night keeper keeps the keep in the town
2 In the big old house in the big old gown
3 The house in the town had the big old keep
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5 The night keeper keeps the keep in the night
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term t	f_t	$L(t)$
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`big`	2	$[2, 3]$
`dark`	1	$[6]$
...	...	...
`had`	1	$[3]$
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`in`	5	$[1, 2, 3, 5, 6]$
...	...	...

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List Encodings

- Δ-encoding
- unary- and ternary-encoding
- Elia-γ and -δ-encoding
- Golomb- and Fibonacci-encoding

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Recap: Pattern Matching with the LCP-Array (1/3)

- remember how many characters of the pattern and suffix match
- identify how long the prefix of the old and next suffix is
- do so using the LCP-array and
- range minimum queries detailed introduction in **Advanced Data Structures**

- $lcp(i, j) = \max\{k \colon T[i..i + k)$
- $lcp(i, j) = T[j..j+k)\} = LCP[RMQ_{LCP}(i+1, j)]$
- RMQs can be answered in $O(1)$ time and
- require $O(n)$ space

Definition: Range Minimum Queries

Given an array $A[1..m)$, a **range minimum query** for a range $\ell \leq r \in [1, n)$ returns

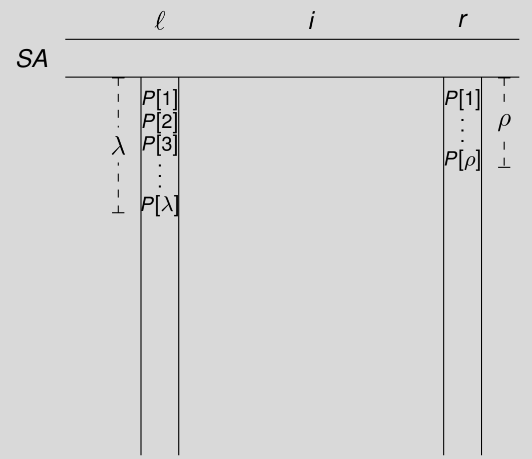
$$RMQ_A(\ell, r) = \arg\min\{A[k] \colon k \in [\ell, r]\}$$

Recap: Pattern Matching with the LCP-Array (2/3)

- during binary search matched
- λ characters with left border ℓ and
- ρ characters with right border r
- w.l.o.g. let $\lambda \geq \rho$

- middle position i
- decide if continue in $[\ell, i]$ or $[i, r]$

- let $\xi = lcp(SA[\ell], SA[i])$ ℹ $O(1)$ time with RMQs



Recap: Pattern Matching with the LCP-Array (3/3)

- let $\xi = lcp(SA[\ell], SA[i])$

$\xi > \lambda$

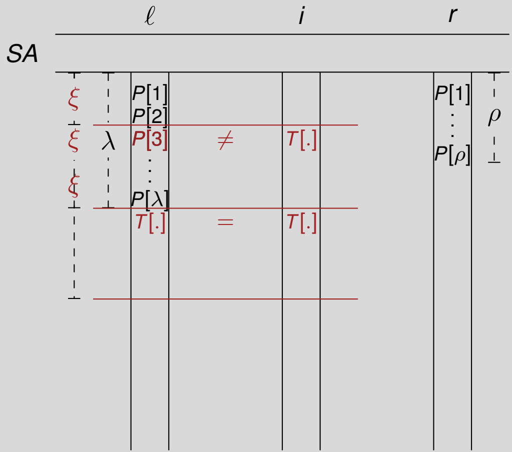
- $P[\lambda + 1] > T[SA[\ell] + \lambda] = T[SA[i] + \lambda]$
- $\ell = i$ without character comparison

$\xi = \lambda$

- compare as before

$\xi < \lambda$

- $\xi \geq \rho$ and $P[\xi + 1] < T[SA[i] + \xi]$
- $r = i$ and $\rho = \xi$ without character comparison



Old Problem, New Name

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Definition: Longest Common Extensions

Given a text T of size n over an alphabet of size σ, construct data structure that answers for $i, j \in [1, n]$

$$\mathrm{lce}_T(i, j) = \max\{\ell \geq 0 \colon T[i, i+\ell) = T[j, j+\ell)\}$$

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	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
T	A	B	C	D	A	B	C	C	D	B	C	C	B	A	B	C	D	A	D	A

$\mathrm{lce}_T(1, 14) = 0\ 1\ 2\ 3\ 4\ 5$

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Applications

- (sparse) suffix sorting
- approximate pattern matching
- ...

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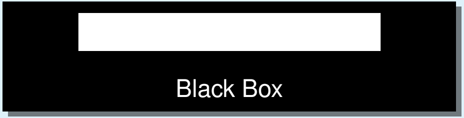
Practical Algorithms for Longest Common Extensions [IT09]

Sophisticated Black Box (BB)

- based on ISA, LCP, and RMQ



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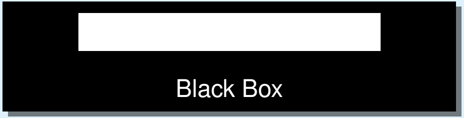

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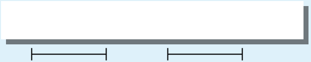
- compare character by character



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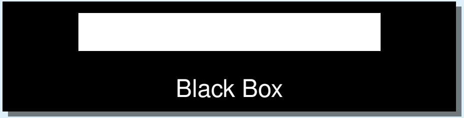

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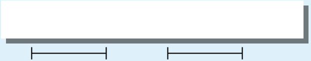

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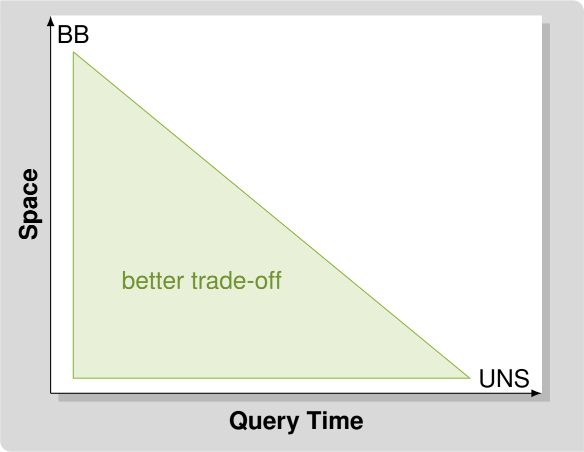

Monte Carlo and Las Vegas Algorithms

- setting: randomized algorithms

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- some Monte Carlo algorithms can be turned into Las Vegas algorithms
- depends on correctness check
- all Monte Carlo algorithms presented today can be turned into Las Vegas algorithms

Randomized String Matching

- compute s of strings
- application not limited to LCEs

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Definition: Karp-Rabin Fingerprint [KR87]

Given a text T of length n over an alphabet of size σ and a random prime number $q \in \Theta(n^c)$, the Karp-Rabin fingerprint of $T[i..j]$ is

$$(i, j) = (\sum_{k=i}^{j} T[k] \cdot \sigma^{j-k}) \bmod q$$

ℹ $(x + y) \bmod z = z \bmod z + y \bmod z \pmod{z}$

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- if $T[i..i+\ell] = T[j..j+\ell]$, then

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- if $T[i..i+\ell] \neq T[j..j+\ell]$, then

$$\text{Prob}((i, i+\ell) = (j, j+\ell)) \in O\left(\frac{\ell \lg \sigma}{n^c}\right)$$

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- example on the board

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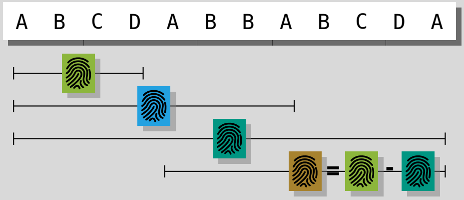

- overwrite text with fingerprints (in-place)

- all parts of text are restorable
- how?

Overwriting the Text with Fingerprints (2/2)

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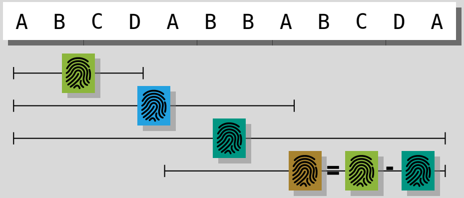

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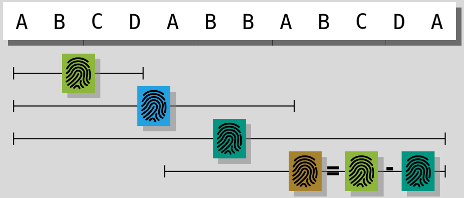

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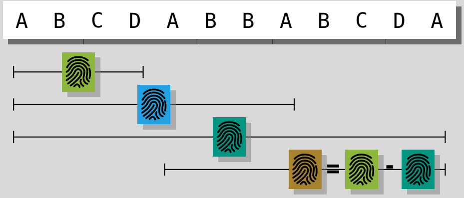

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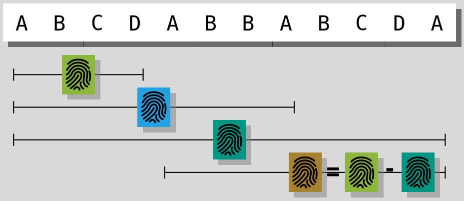

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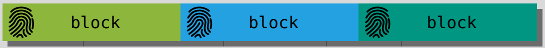

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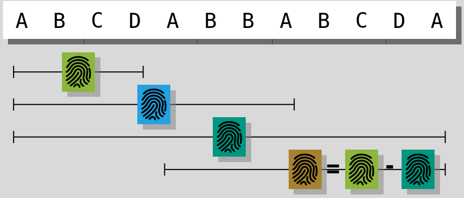

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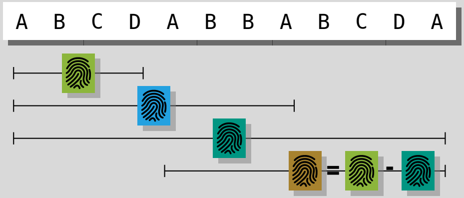

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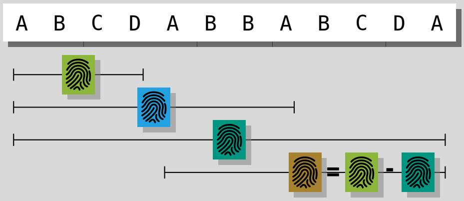

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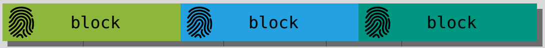

- enough to answer LCE queries

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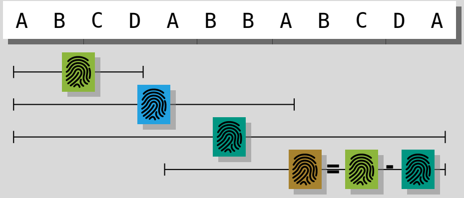
- this gives us access to the text(!)

- q can be chosen such that MSB of $P'[i]$ is zero w.h.p., then
- D can be stored in the MSBs 



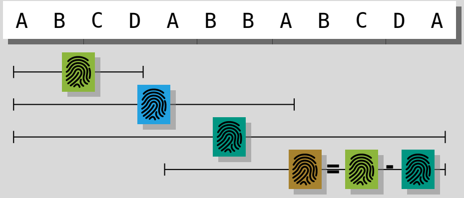
- overwrite text with fingerprints (in-place)

- enough to answer LCE queries
- how?

Answering LCE Queries with Fingerprints

LCEs with Fingerprints

- compute LCE of i and j
- exponential search until $(i, i + 2^k) \neq (j, j + 2^k)$
- binary search to find correct block m
- recompute $B[m]$ and find mismatching character



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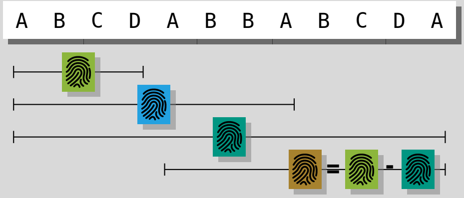
- requires $O(\lg \ell)$ time for LCEs of size ℓ



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String Synchronizing Sets (Simplified, 1/2)

Definition: Simplified τ-Synchronizing Sets [KK19]

Given a text T of length n and $0 < \tau \leq n/2$, a simplified τ-synchronizing set S of T is

$$S = \{i \in [1, n-2\tau+1] \colon \mathit{min}\{\text{🫆}(j, j+\tau-1) \colon j \in [i, i+\tau]\} \in \{\text{🫆}(i, i+\tau-1), \text{🫆}(i+\tau, i+2\tau-1)\}\}$$

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Karlsruhe Institute of Technology

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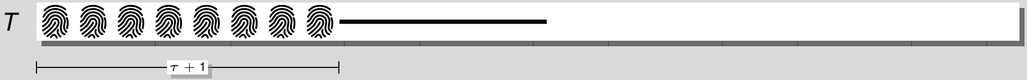

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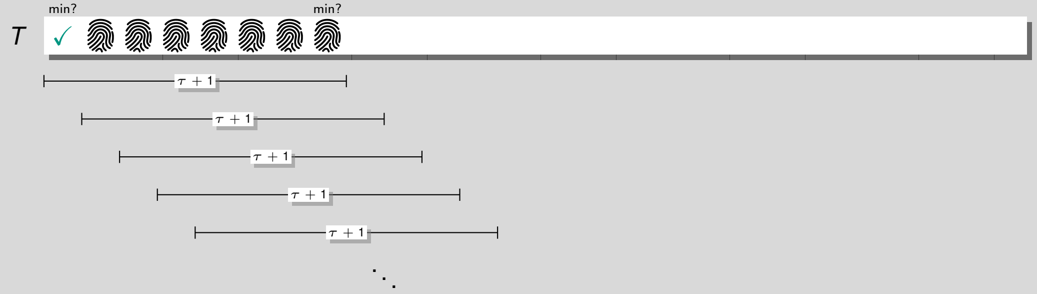


String Synchronizing Sets (Simplified, 2/2)

- $|S| = \Theta(n/\tau)$ in practice (on most data sets)
- more complex definition required to obtain this size

Consistency & (Simplified) Density Property

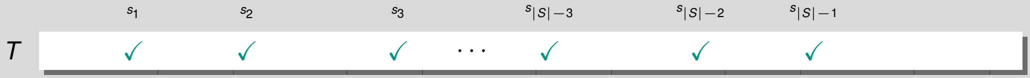
- for all $i, j \in [1, n - 2\tau + 1]$ we have

$$T[i, i+2\tau-1] = T[j, j+2\tau-1] \Rightarrow i \in S \Leftrightarrow j \in S$$

- for any τ consecutive positions there is at least one position in S

Answering LCE Queries with String Synchronizing Sets (1/2)

Text T' for Positions in S



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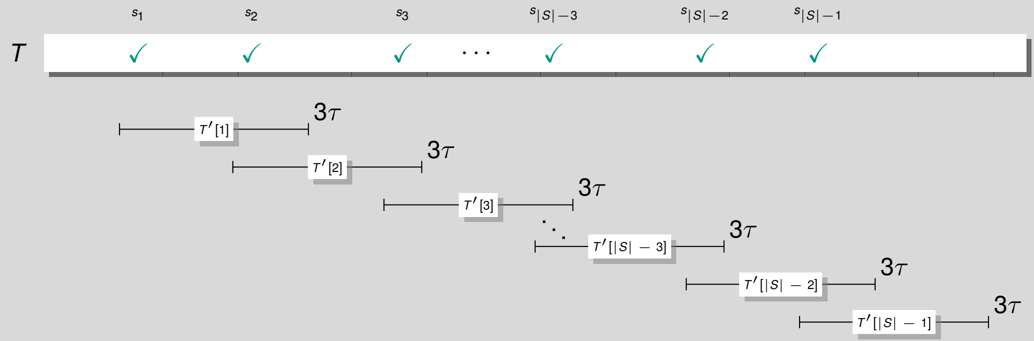

Answering LCE Queries with String Synchronizing Sets (2/2)

- in practice, we sort the substrings
- characters of T' are the ranks of substrings
- build BB LCE for T' w.r.t. length in T

Answering Queries

- compare naively for 3τ characters
- if equal find successors of i and j in S
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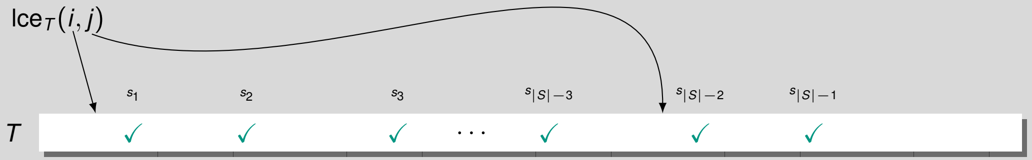

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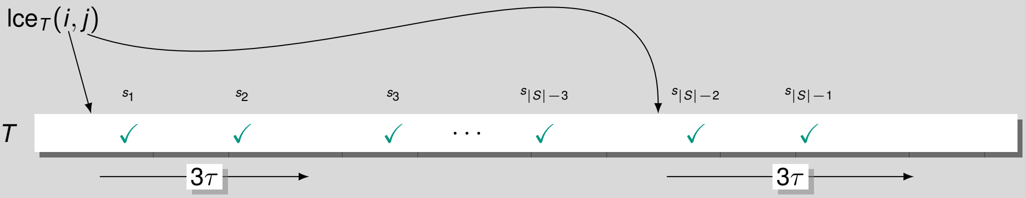

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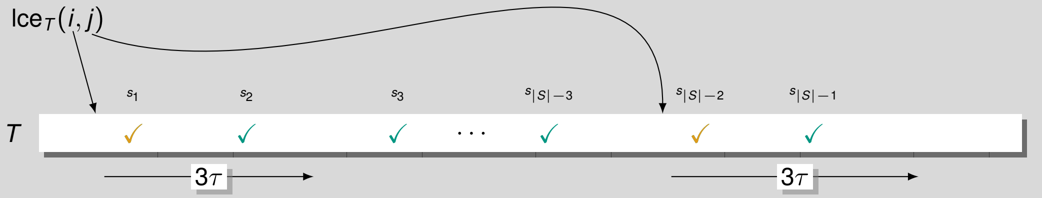

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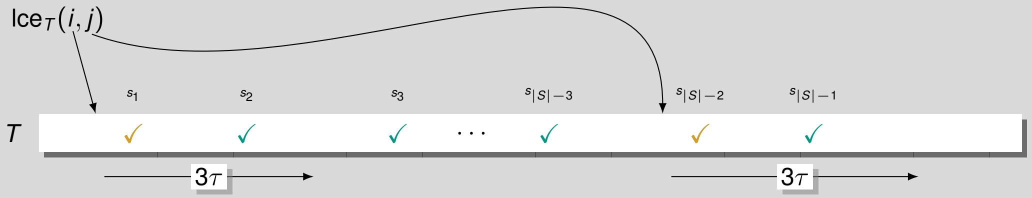

- in this example: $\mathsf{lce}_T(i,j) = s_1 - i + \mathsf{lce}_{T'}(1, |S| - 2)$

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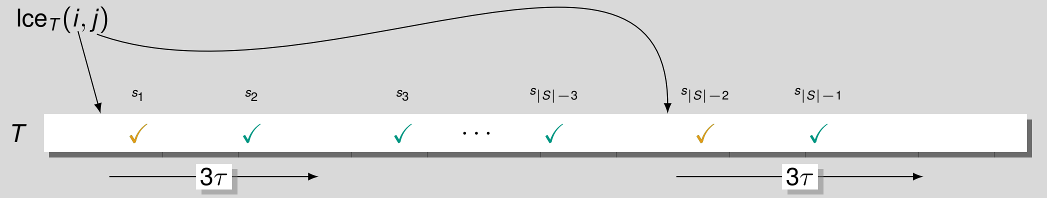
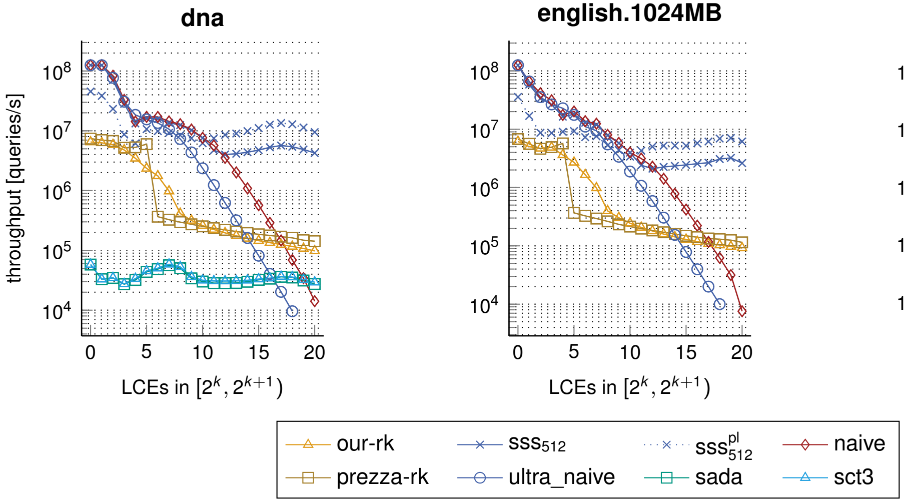


- in this example: $\mathsf{lce}_T(i,j) = s_1 - i + \mathsf{lce}_{T'}(1, |S| - 2)$
- in practice: we have a very fast static successor data structure

Practical Evaluation [Din+20]



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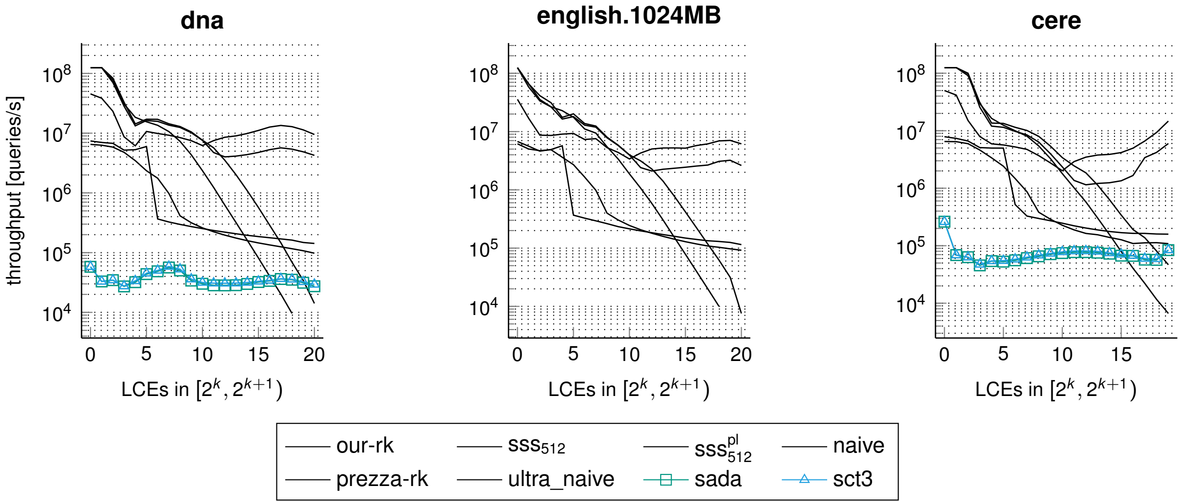

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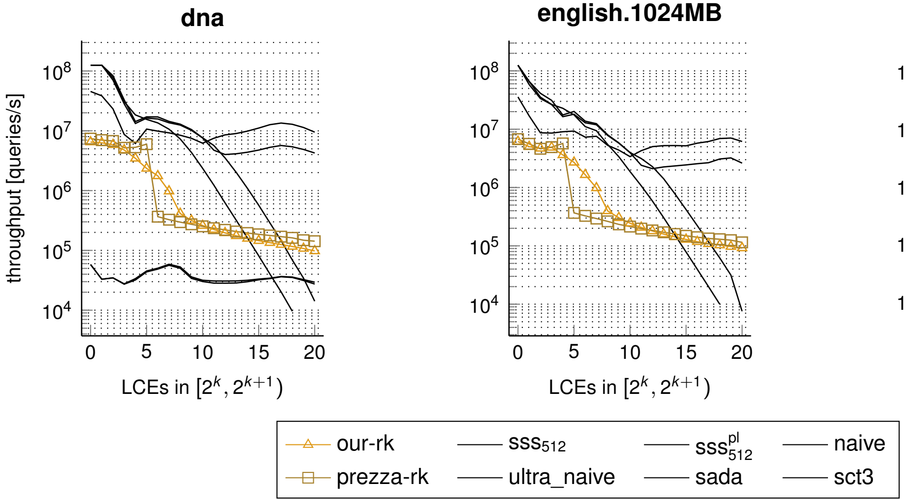

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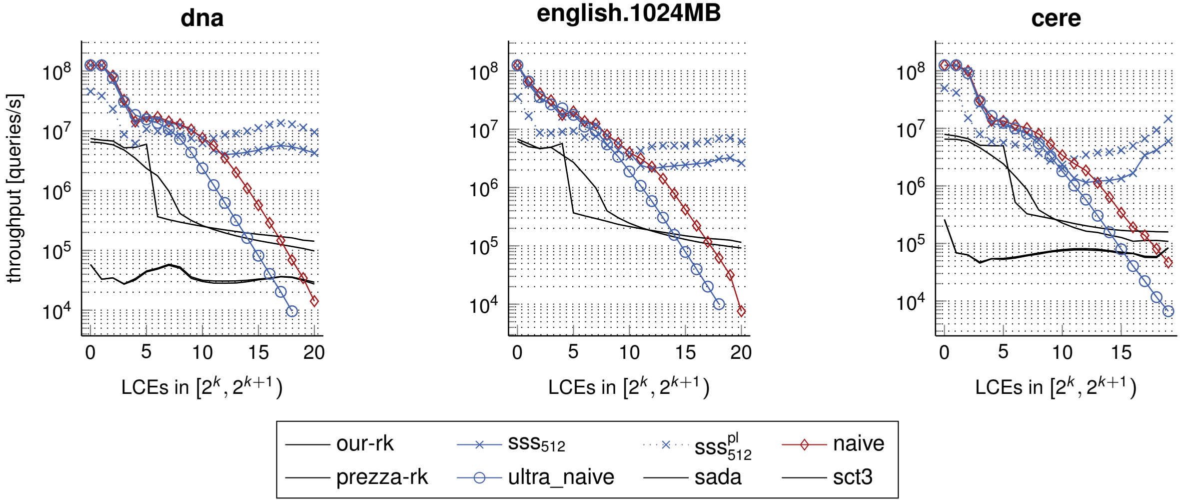

Conclusion and Outlook

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This Lecture

- longest common extension queries
- Karp-Rabin fingerprints
- string synchronizing sets

Thats all! We are (mostly) done.

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Next Lecture

- big recap and Q&A

Thats all! We are (mostly) done.