Karlsruhe Institute of Technology

Text Indexing

Lecture 01: Tries

Florian Kurpicz

PINGO

https://pingo.scc.kit.edu/952701

Preliminaries (1/2)

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Definition: Text

- let Σ be an **alphabet**
- $T \in \Sigma^\star$ is a text
- $|T| = n$ is the length of the string
- $T = T[1]T[2]\ldots T[n]$

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Definition: Alphabet Types

- **constant size alphabet**: finite set not depending on n
- **integer alphabet**: alphabet is $\{1, \dots, \sigma\}$ and fits into constant number of words
- **finite alphabets**: alphabet of finite size

Preliminaries (2/2)

Definition: Substring, Prefix, and Suffix

Given a text $T = T[1]T[2]\dots T[n]$ of length n:

- $T[i..j] = T[i]\dots T[j]$ is called a **substring**,

a	b	b	a	a	b	b	a	$

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Sentinel for Simplicity

Given a text T of length n over an alphabet Σ.

- we assume that $T[n] = \$$ with
- $\$ \notin \Sigma$ and $\$ < \alpha$ for all $\alpha \in \Sigma$

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- otherwise, suffix can be prefix of another suffix

1	2	3	4	5	6	7	8
a	b	b	a	a	b	b	a

- $T[1..n] = \texttt{abbaabba}$ and $T[5..n] = \texttt{abba}$

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- $T[1..n] = \texttt{abbaabba}$ and $T[5..n] = \texttt{abba}$

Definition: Prefix-Free

A string is **prefix-free** if no suffix is a prefix of another suffix

String Dictionary

Given a set $S \subseteq \Sigma^\star$ of prefix-free strings, we want to answer:

- is $x \in \Sigma^\star$ in S
- add $x \notin S$ to S
- remove $x \in S$ from S
- predecessor and successor of $x \in \Sigma^\star$ in S

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Given a set $S = \{S_1, \dots, S_k\}$ of prefix-free strings, a trie is a labeled rooted tree $G = (V, E)$ with:

1. k leaves
2. $\forall S_i \in S$ there is a path from the root to a leaf, such that the concatenation of the labels is S_i
3. $\forall v \in V$ the labels of the edges $(v, \cdot)$ are unique

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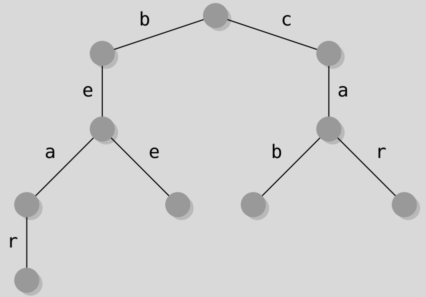


$S = \{\texttt{bear}, \texttt{bee}, \texttt{cab}, \texttt{car}\}$

Queries: Insert, Contains, and Delete a Pattern

Same for all

- start at root and follow existing children

Contains

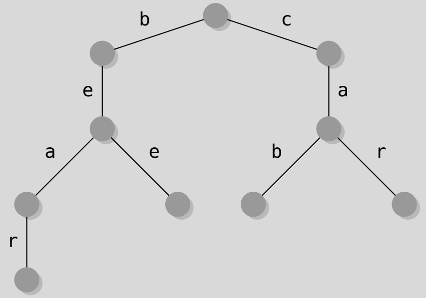
- is leaf found and whole pattern is matched

Delete

- if leaf is found backtrack and delete unique path
  ℹ otherwise not found

Insert

- insert rest of pattern ℹ prefix-free



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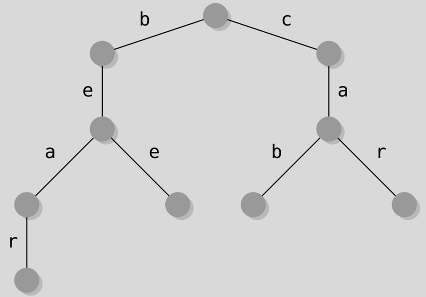

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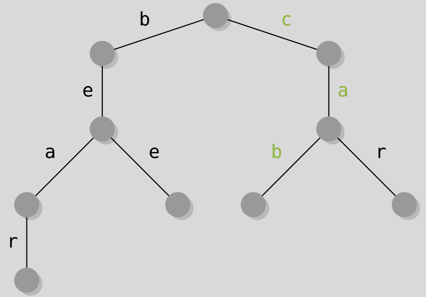

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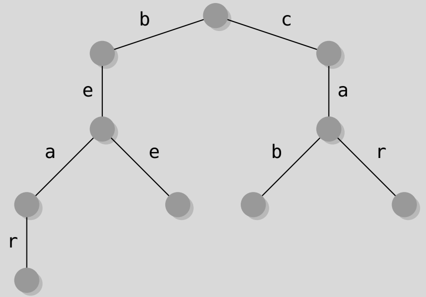

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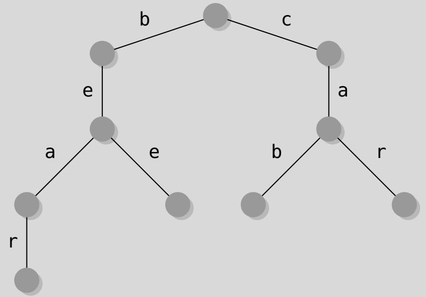

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- is cab in S
- remove bear from S

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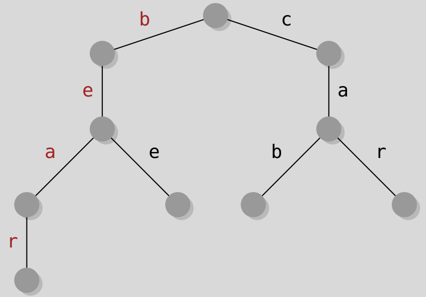

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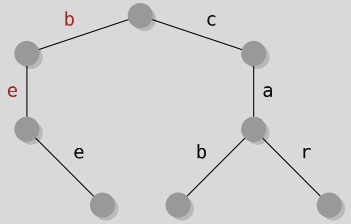

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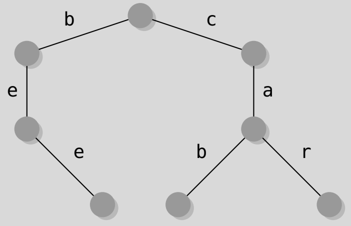

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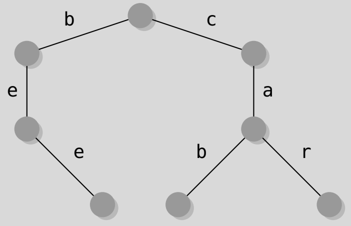

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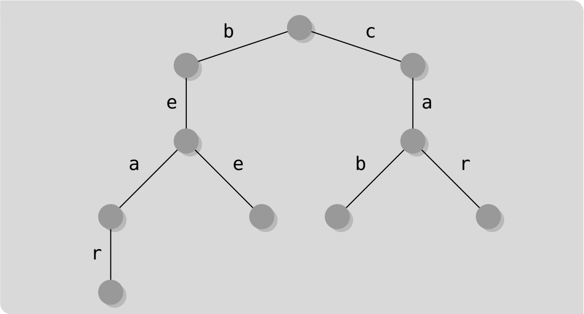
- is cab in S
- remove bear from S
- how can we find the predecessor of can?

Why Prefix-Free

- insert `beer`



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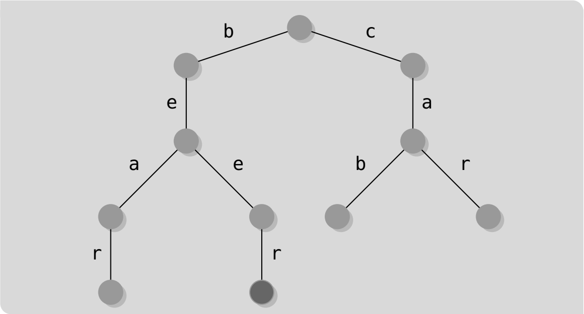

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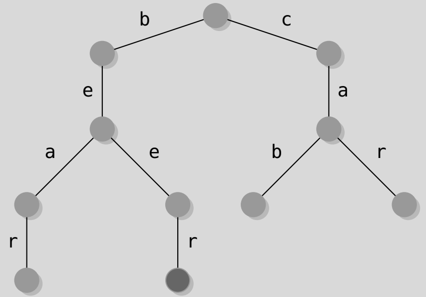

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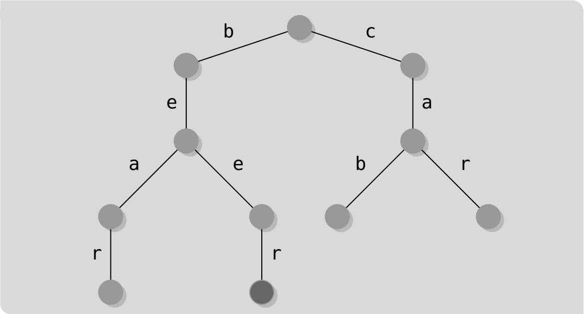
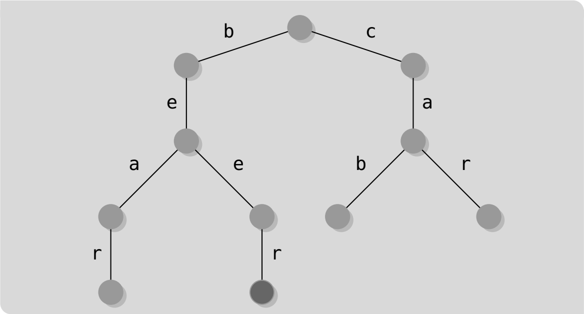


Why Prefix-Free

- insert `beer`
- `bee` cannot be found
- remember which node refers to a string
- or (much preferred) make strings prefix free



Next Steps

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Setting

- alphabet Σ of size σ
- k strings $\{s_1, \ldots, s_k\}$ over the alphabet Σ
- total size of strings is $N = \sum_{i=1}^{k} |s_i|$
- queries ask for pattern P of length m

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- query times
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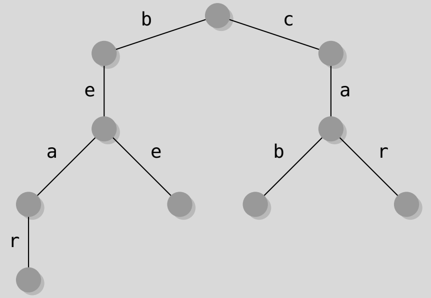

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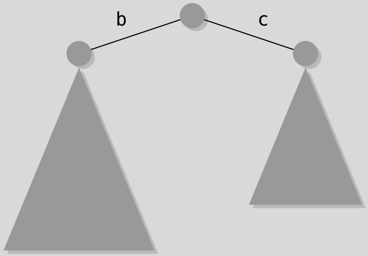

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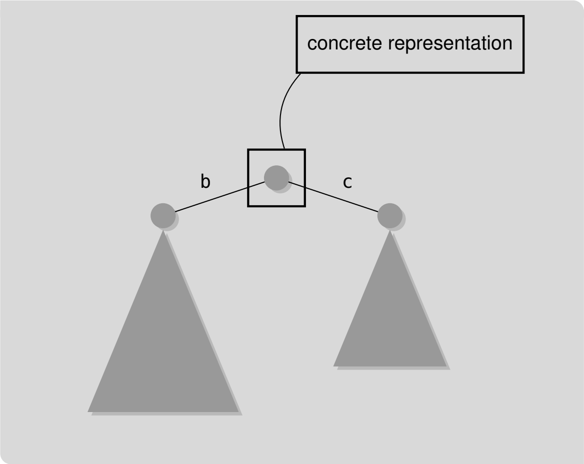
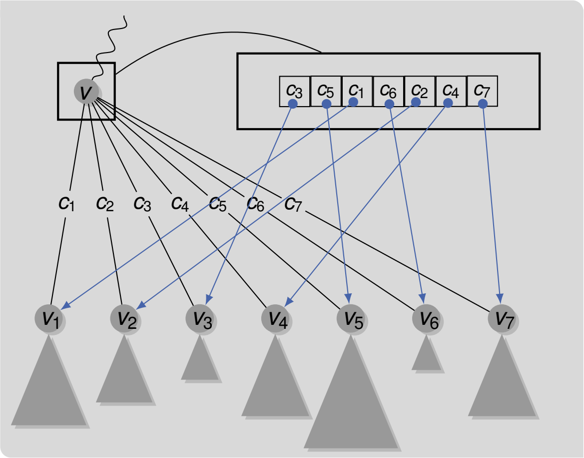


Arrays of Variable Size

- store children (character and pointer) in the order they are added
- to find child scan array
- to delete child swap with last and remove last
  - children are not ordered
- **PINGO** query time?



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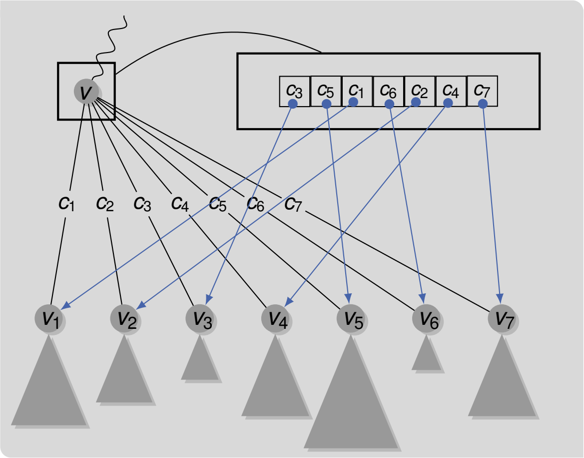

Query Time (Contains)

- $O(m \cdot \sigma)$

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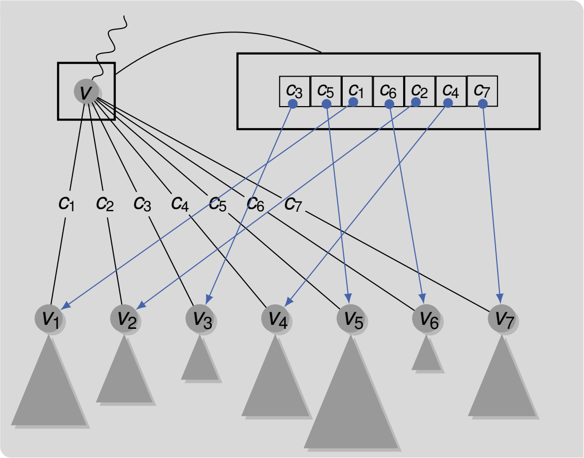
Space

- $O(N)$ words



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- children (pointer) are stored in arrays of size σ
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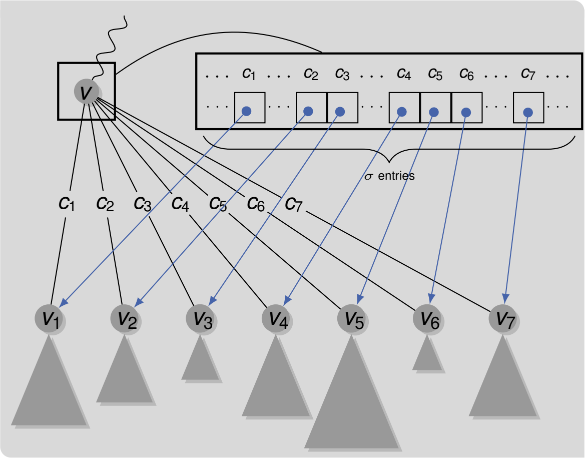

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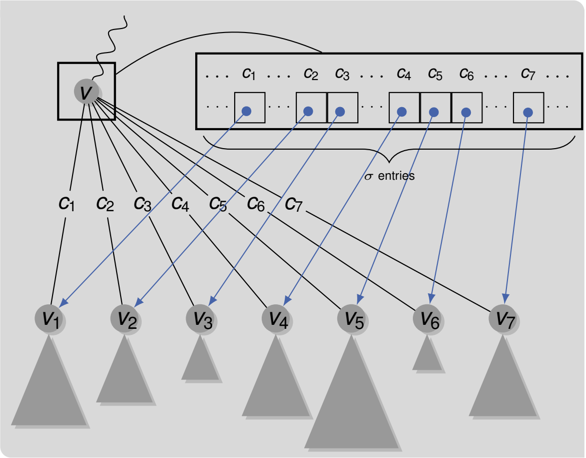

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Query Time (Contains)

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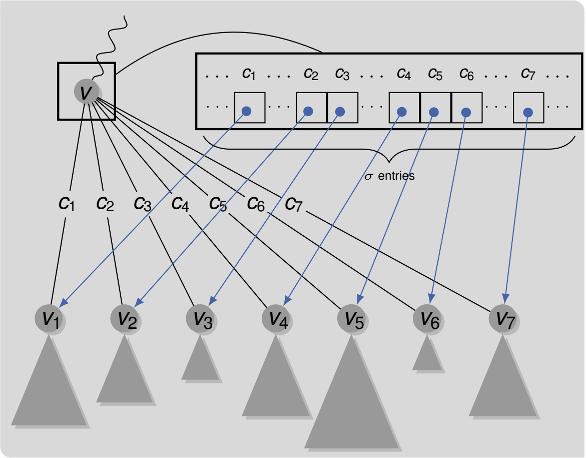
Space

- $O(N \cdot \sigma)$ words ℹ very bad



Hash Tables

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  - has overhead
- or use global hash table for whole trie
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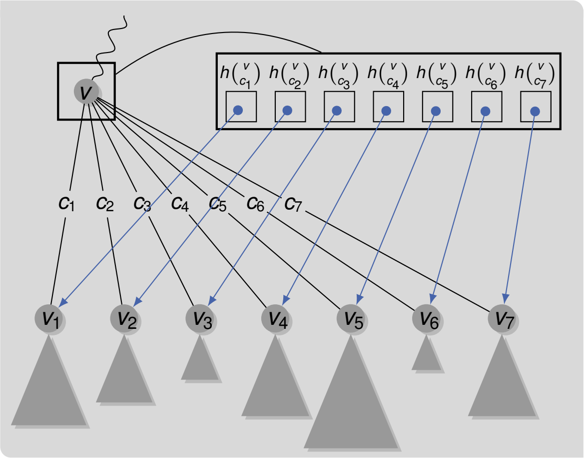

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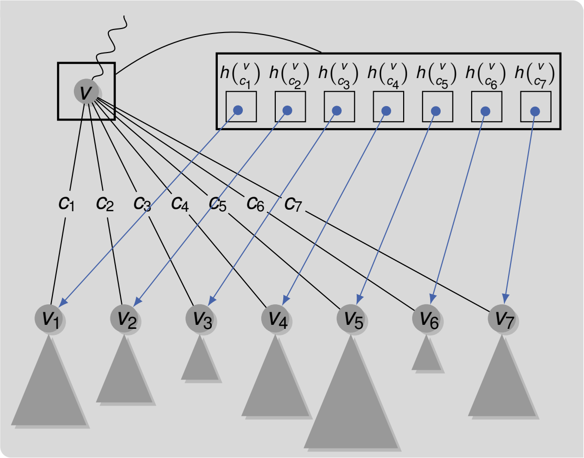

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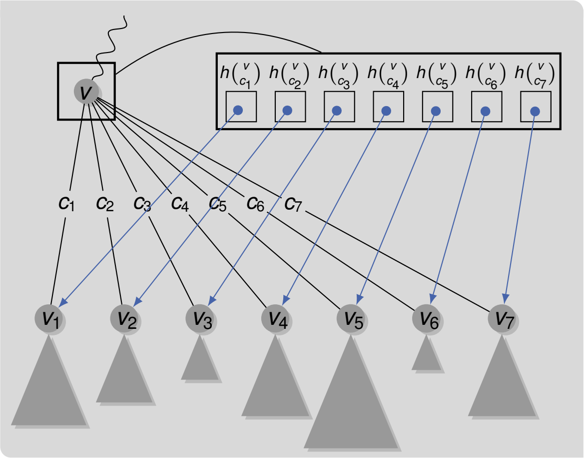
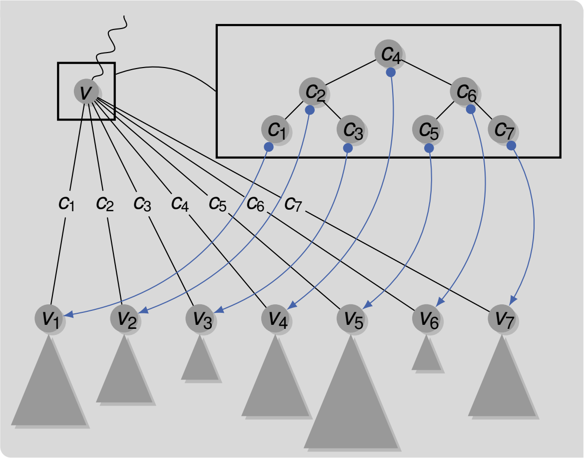


Balanced Search Trees

- children are stored in balanced search trees
- e.g., AVL tree, red-black tree, . . .
- in static setting sorted array and binary search
- **PINGO** query time?



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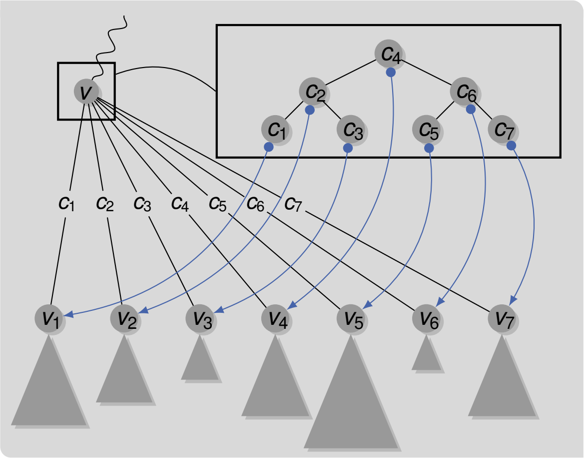

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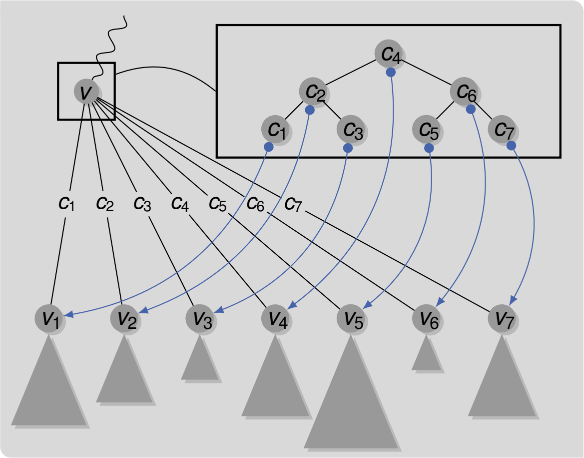
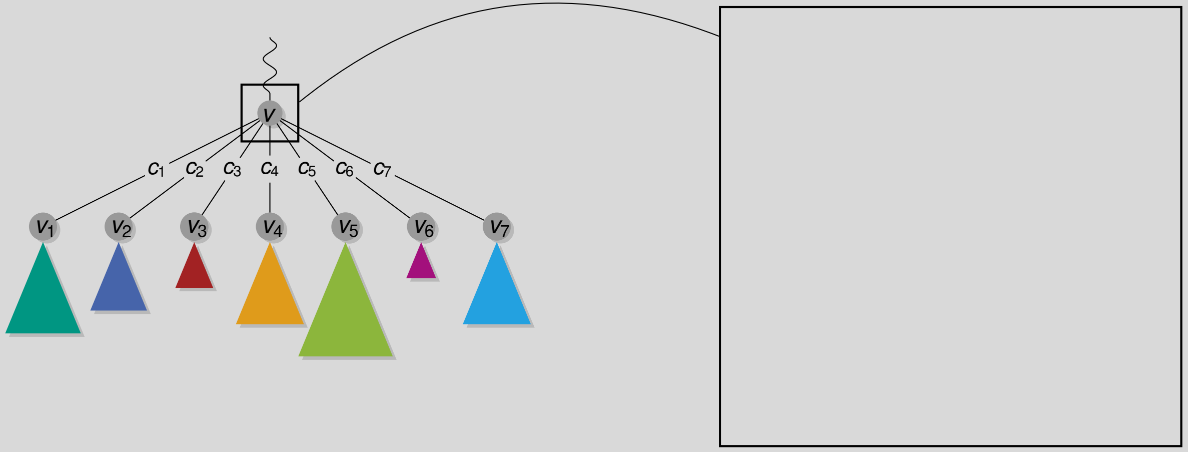


Weight-Balanced Search Trees (1/2)



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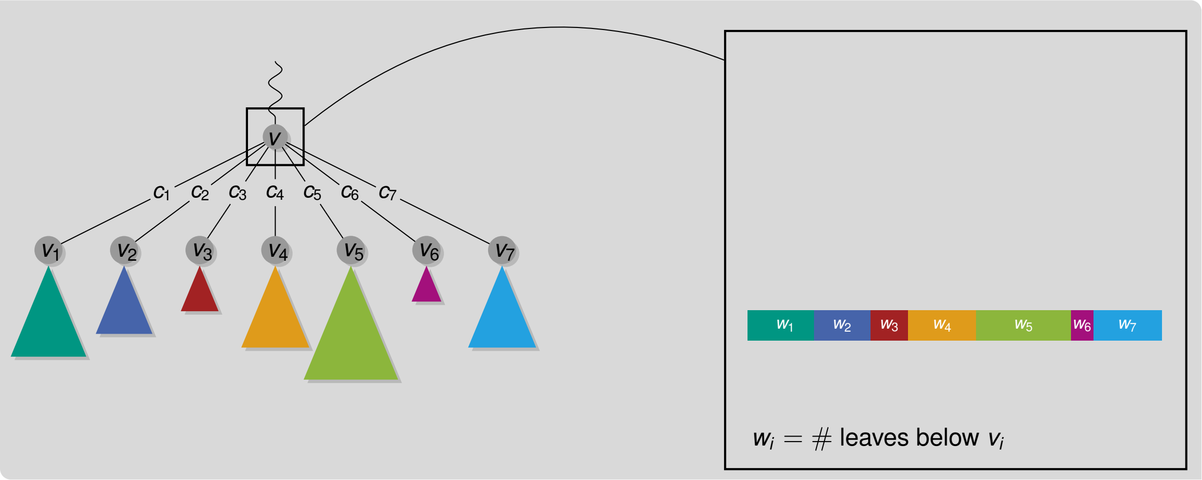

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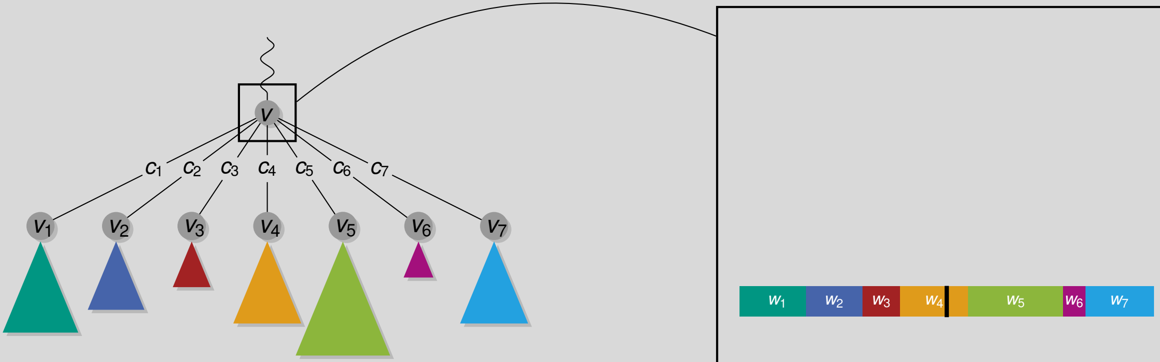

$w_i = \#$ leaves below v_i

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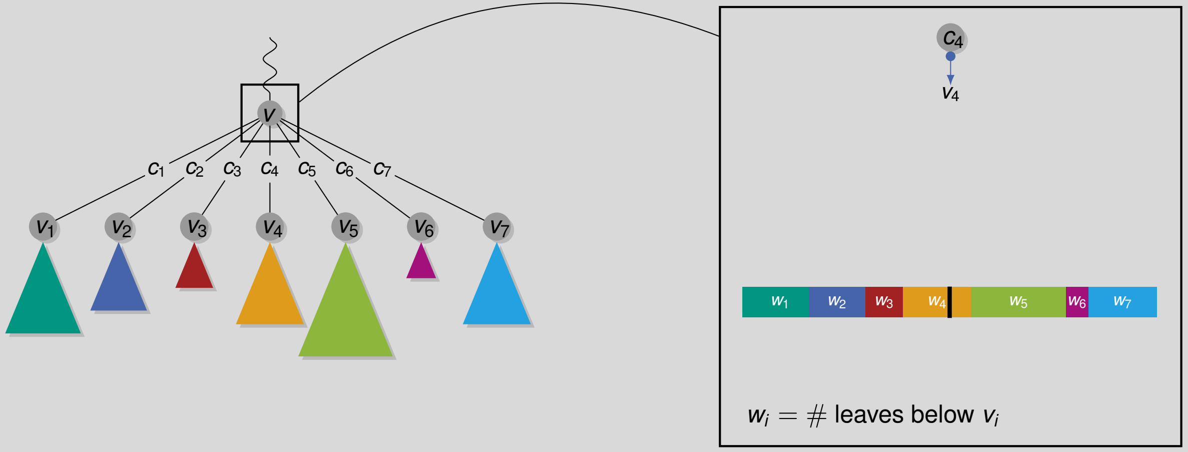

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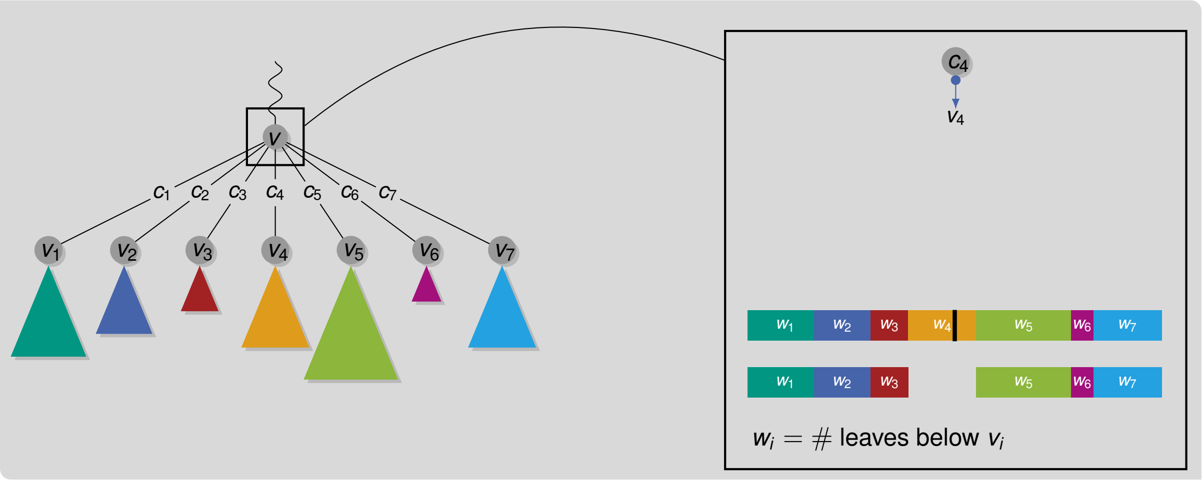

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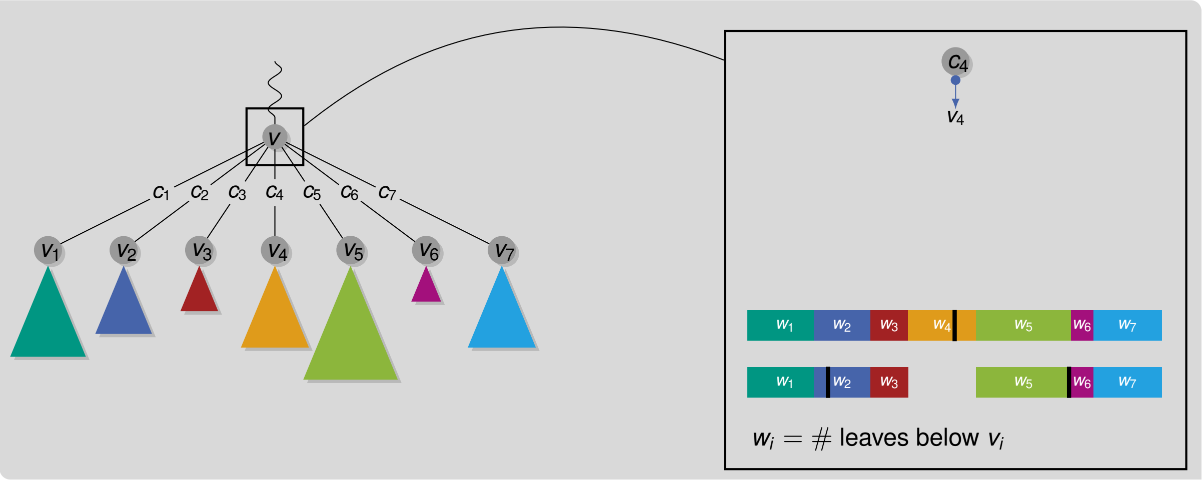

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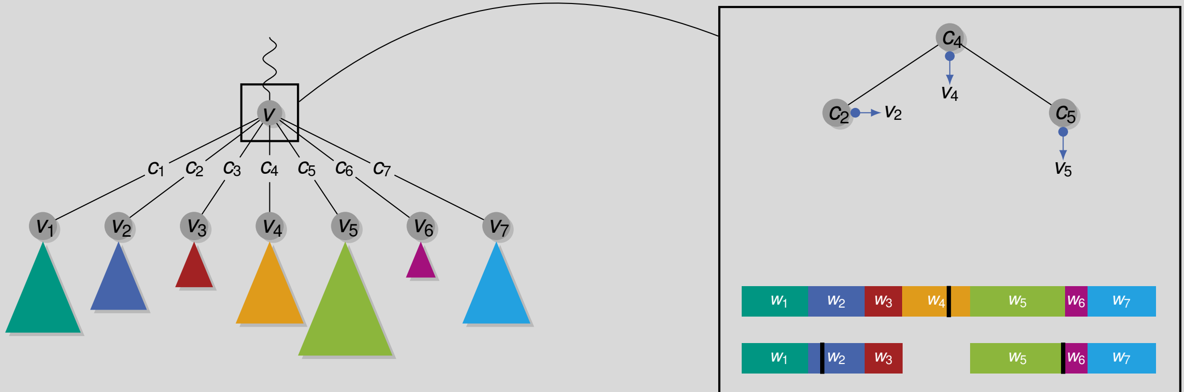

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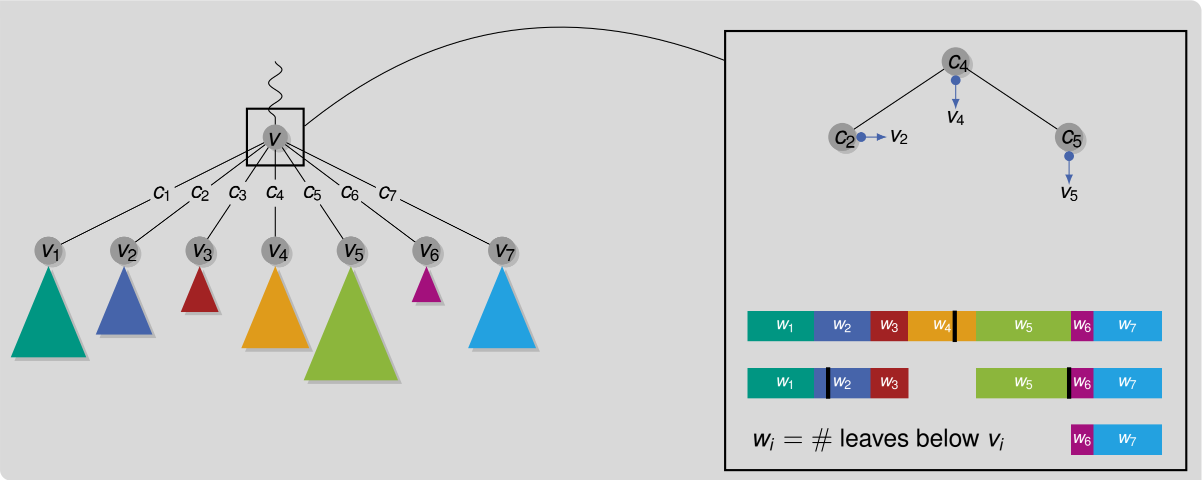

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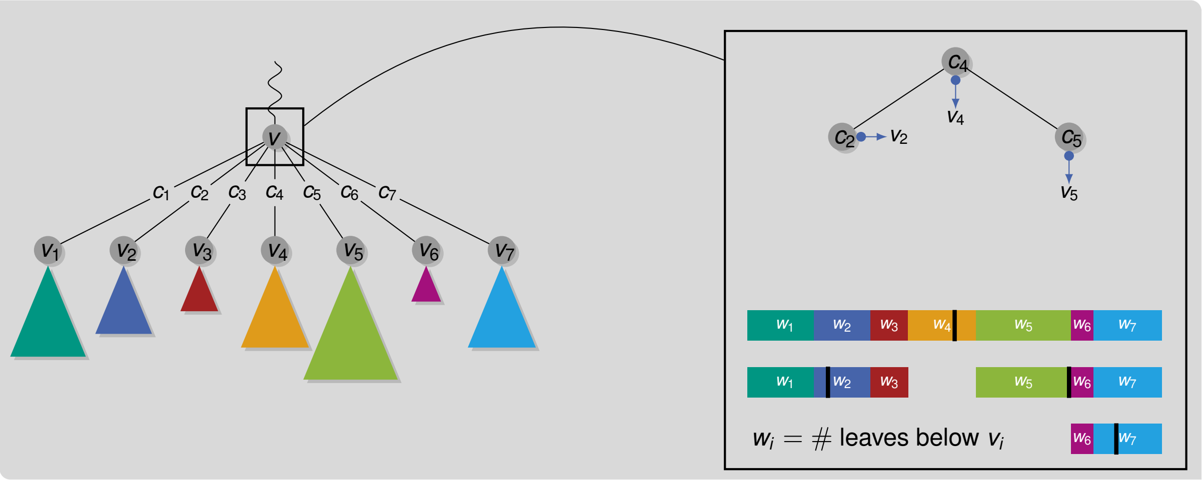

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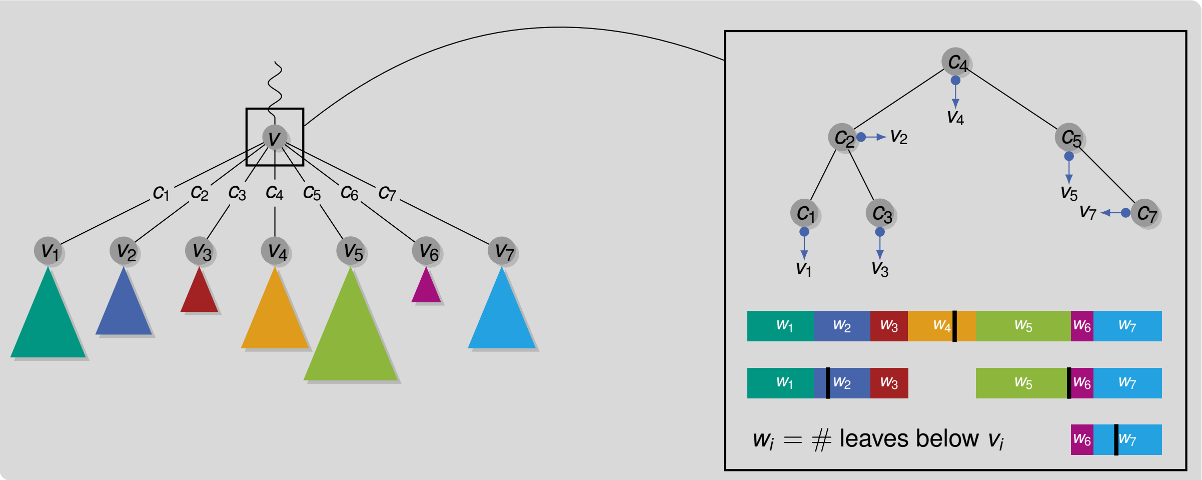

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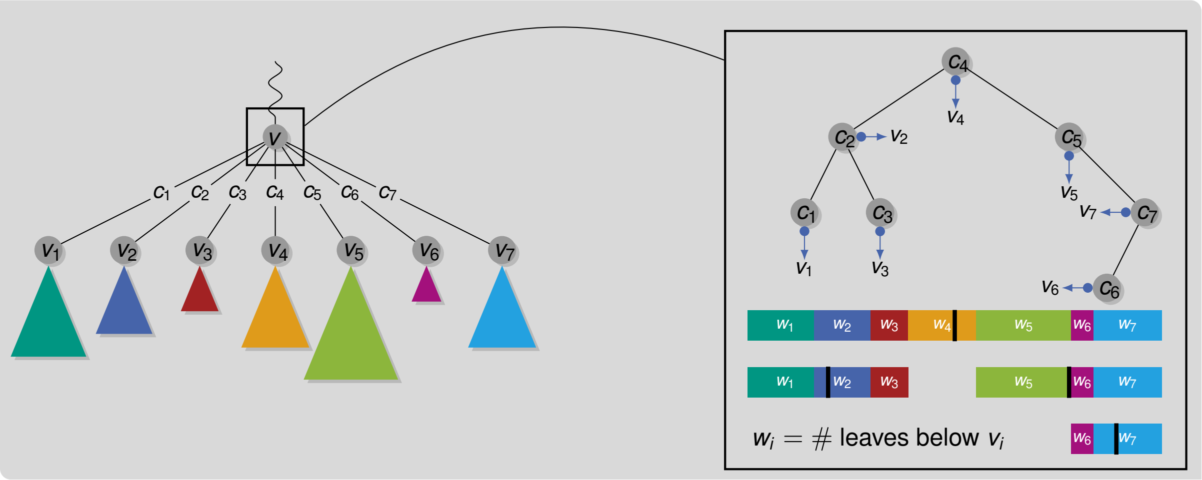
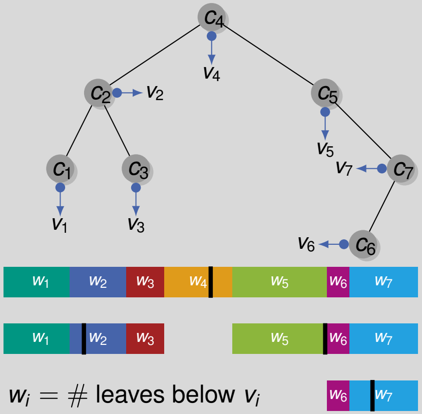


Weight-Balanced Search Trees (2/2)

- use weight-balanced search trees at each node
- **PINGO** query time?



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Weight-Balanced Search Trees (2/2)

- use weight-balanced search trees at each node
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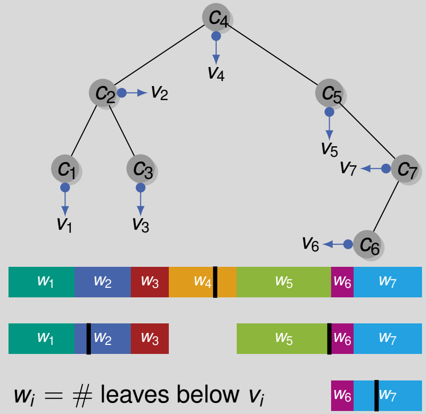
Query Time (Contains)

- $O(m + \lg k)$
- match character of pattern
- or halve number of strings



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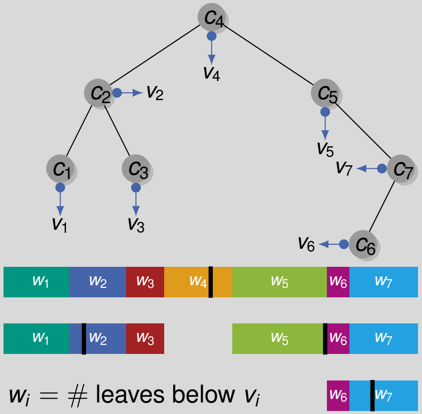
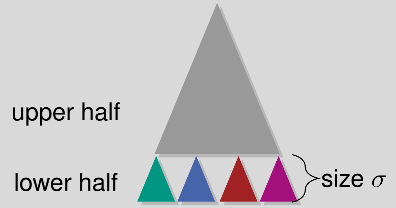
Space

- $O(N)$ words



Two-Levels with Weight-Balanced Search Trees

- split tree into upper and lower half
- lower half deepest nodes such that subtrees have size $O(\sigma)$
- weight-balanced search trees for lower half
- fixed-size arrays in upper half ℹ branching nodes only
- **PINGO** query time?



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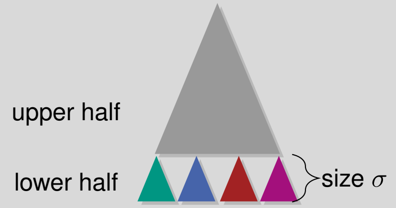

Query Time (Contains)

- upper half: $O(m)$
- lower half: $O(m + \lg \sigma)$
- total: $O(m + \lg \sigma)$

Two-Levels with Weight-Balanced Search Trees

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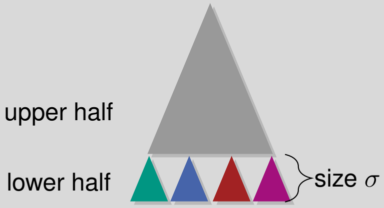


Query Time (Contains)

- upper half: $O(m)$
- lower half: $O(m + \lg \sigma)$
- total: $O(m + \lg \sigma)$

Space

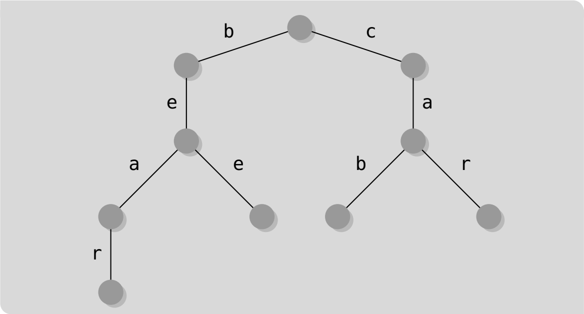
- upper half: $O(N)$ words
  ℹ $O(N/\sigma)$ branching nodes
- lower half: $O(N)$ words
- total: $O(N)$ words

Theoretical Comparison

Representation	Query Time (Contains)	Space in Words
arrays of variable size	$O(m \cdot \sigma)$	$O(N)$
arrays of fixed size	$O(m)$	$O(N \cdot \sigma)$
hash tables	$O(m)$ w.h.p.	$O(N)$
balanced search trees	$O(m \cdot \lg \sigma)$	$O(N)$
weight-balanced search trees	$O(m + \lg k)$	$O(N)$
two-levels with weight-balanced search trees	$O(m + \lg \sigma)$	$O(N)$

Compact Trie

- tries have unnecessary nodes
- branchless paths can be removed
- edge labels can consist of multiple characters



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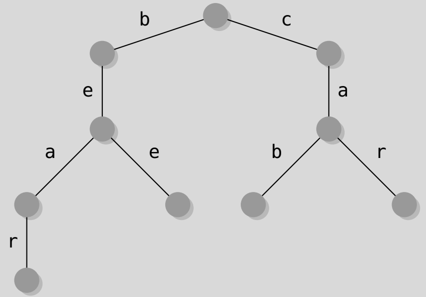

Definition: Compact Trie

- A compact trie is a trie where all branchless paths are replaced by a single edge.
- The label of the new edge is the concatenation of the replaced edges' labels.

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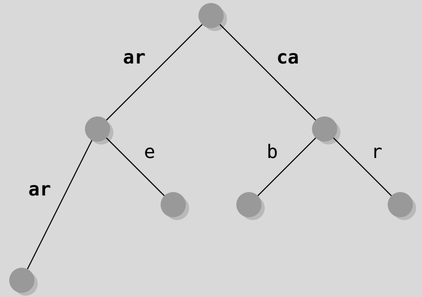

Conclusion and Outlook

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This Lecture

- dictionaries
- tries with different space-time trade-off

Conclusion and Outlook

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Next Lecture

- suffix trees and suffix arrays
- no lecture on Halloween(!)
- next lecture 07.11.2022